B. Tech Degree I & II Semester Examination in Marine Engineering June 2012

MRE 101 ENGINEERING MATHEMATICS I

(All questions carry ***Equal*** marks)

Time : 3 Hours | Maximum Marks :100

I. (a) State mean value theorem and use it to obtain the point C in the interval $(-2,1)$ for the function $f(x) = x - x^3$.

(b) Evaluate $\underset{x \to 0}{Lt} \dfrac{a^x - 1}{b^x - 1}$.

OR

II. (a) Find the radius of curvature of $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 2$ at $(3,4)$

(b) If $y = e^{m\sin^{-1}x}$, prove that $(1-x^2)y_2 - xy_1, -m^2y = 0$. Differentiate the above equation n times with reference to x by using Leibnitz formula.

III. (a) Verify Euler's theorem for the function $u = x^n \sin(y/x)$.

(b) If $z = f(x^2 + y^2)$, show that $x\dfrac{\partial z}{\partial y} - y\dfrac{\partial z}{\partial x} = 0$.

OR

IV. (a) If $u = \sin^{-1}\left(\dfrac{x^2 + Y^2}{x + y}\right)$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$.

(b) Show that the function $f(x,y) = x^3 + y^3 - 6\ 3(x+y) + 12xy$ has a maximum at $(-7, -7)$ and a minimum at $(3, 3)$.

V. (a) Show that the locus of the point of intersection of two perpendicular tangents to a parabola is its directrix.

(b) Find the equation of the asymptotes of the hyperbola $2x^2 - xy + 3y^2 - 9x + 16y - 8 = 0$.

OR

VI. (a) Derive standard equation of hyperbola.

(b) Find the eccentricity, foci, directrices and length of the lactus rectum of the ellipse $9x^2 + 25y^2 - 18x - 100y - 116 = 0$.

(P.T.O.)

VII. (a) Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2 yz \; dx\,dy\,dz$.

(b) Find the volume of a sphere of radius a, using spherical polar co-ordinates.

OR

VIII. (a) Find the area of the cardiod $r = a(1-\cos\theta)$.

(b) Evaluate $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x+y+z)\,dxdydz$.

IX. (a) Prove that $\text{div}(grad\, r^n) = n(n+1)r^{n-2}$.

(b) Show that the vector point function $\overline{F} = grad\,(x^3 + y^3 + z^3 - 3xyz)$ forms a conservative field.

OR

X. (a) If $\overline{a}$ and $\overline{b}$ are constant vectors, prove that

(i) $div\left[(\overline{r} \times \overline{a}) \times \overline{b}\right] = -2\overline{b} \cdot \overline{a}$

(ii) $curl\left[(\overline{r} \times \overline{a}) \times \overline{b}\right] = \overline{b} \times \overline{a}$

(b) If $\overline{f} = (x^2 - yz);-(y^2 - zx)\,j - (z^2 - xy)\,k$, show that grad $(\text{div}\, f) = curl\; curl\; \overline{f} + \nabla^2 f$.

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